

**LEARNING AND TEACHING MATHEMATICS:  
INTERPRETING STUDENT TEACHERS' VOICES**

**SANDRA RUTH SCHUCK**

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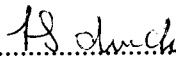
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## **CERTIFICATE**

I certify that this thesis has not already been submitted for any degree and is not being submitted as part of candidature for any other degree.

I also certify that the thesis has been written by me and that any help that I have received in preparing this thesis, and all sources used, have been acknowledged in this thesis.

Signature of Candidate

.....

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## PREFACE

The following publications and conference papers developed from aspects of the research contained in this thesis:

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## **ABSTRACT**

This research study has investigated the beliefs that prospective primary school teachers hold about the epistemology of mathematics, and the teaching and learning of mathematics. In particular, it considered the following questions:

- What beliefs and attitudes about mathematics and mathematics education do first year primary school student teachers bring into their tertiary education?
- Are any of the students' beliefs about mathematics and mathematics education similar to the beliefs of the teacher educators in mathematics education and how do students interact with first year mathematics education subjects in the teacher education course?
- How do students' attitudes and beliefs influence their success in learning new mathematics at this stage of their lives?
- How do students' beliefs and attitudes affect their ideas on good practice in the teaching of mathematics in the primary school?

The research design was qualitative, using a case study investigation of 50 students in their first year of a teacher education course. The students' passage through the first year mathematics education subjects provided valuable insights into their beliefs, principally by means of interviews and open-ended questionnaires. The study was designed to have pedagogical outcomes for the students, by embedding the collection and interpretation of data in the teaching and learning of their course.

My personal perspective throughout this research has been that mathematics is a socio-cultural phenomenon, and that the learning of mathematics is achieved through the mediation of language, social interaction and culture. This perspective of mathematics and the learning of mathematics has influenced the choice of methodology and the research questions asked.

Results indicated that students often held two or more philosophies of mathematics and moved between these philosophies, depending on context. Further, students generally considered that the characteristics of a good teacher included being supportive and enthusiastic. Good pedagogy was believed to incorporate practical activities

demonstrating relevance, and providing “fun” for pupils. However, an alarming result was that having higher order knowledge about mathematics was often seen by the students as being a disadvantage for a teacher, principally because students believed such teachers would be less empathetic to struggling pupils.

These beliefs affected students’ interactions with the first year university mathematics education subjects, as their beliefs about the importance of subject matter knowledge were at variance with the beliefs of the teacher educators. This dissonance led to devaluing of the mathematics education subjects by some of the students.

The study has led to the conclusion that a number of the students’ beliefs about mathematics, and the teaching and learning of mathematics, should not be left unchallenged. Those beliefs dealing with ideas on good pedagogy should be strengthened, while beliefs about the nature of mathematics and the value of subject matter knowledge should be made more transparent and addressed. On the other side of the coin, teacher educators need to acknowledge the differences in the beliefs that student teachers and teacher educators might hold, and to consider ways of making mathematics education courses more relevant and meaningful for students.